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First we express the electron annihilation operators in terms of the Bogoliubov ones, then we evaluate the exp. value.

$\boxed{\hat{c}_{k\uparrow}}$

Look at (7) $-\frac{v_k^*}{u_k}$ (8):

$$\hat{\gamma}_{k\uparrow} - \frac{v_k^*}{u_k}\hat{\gamma}^\dagger_{-k\downarrow} = u_k^*\hat{c}_{k\uparrow} + \cancel{v_k^*\hat{c}^\dagger_{-k\downarrow}} - \cancel{\frac{v_k^*}{u_k}u_k\hat{c}^\dagger_{-k\downarrow}} + \frac{v_k^*}{u_k}v_k\hat{c}_{k\uparrow}$$

$$= \left[u_k^* + \frac{v_k^*}{u_k}v_k\right]\hat{c}_{k\uparrow}$$

Multiply both sides by u_k:

$$u_k\hat{\gamma}_{k\uparrow} - v_k^*\hat{\gamma}^\dagger_{-k\downarrow} = \left[u_k^*u_k + v_k^*v_k\right]\hat{c}_{k\uparrow} = \hat{c}_{k\uparrow}$$

$|u_k|^2 + |v_k|^2 = 1$

$\boxed{\hat{c}_{-k\downarrow}}$ Take (7) and (8), change $k\to -k$, $\uparrow\to\downarrow$.

Note the changes of sign in the second terms of the RHS.

(I) $\hat{\gamma}_{-k\downarrow} = u_k^*\hat{c}_{-k\downarrow} - v_k^*\hat{c}^\dagger_{k\uparrow}$

(II) $\hat{\gamma}^\dagger_{k\uparrow} = u_k\hat{c}^\dagger_{k\uparrow} + v_k\hat{c}_{-k\downarrow}$

and look at (I) $+ \frac{v_k^*}{u_k}$ (II)

$$\hat{\gamma}_{-k\downarrow} + \frac{v_k^*}{u_k}\hat{\gamma}^{\dagger}_{k\uparrow} = \left[u_k^* + \frac{v_k^*}{u_k} v_k\right]\hat{c}_{-k\downarrow}$$

ignoring the terms that drop by construction of the sum.

Now multiply both sides by u_k to yield:

$$u_k \hat{\gamma}_{-k\downarrow} + v_k^* \hat{\gamma}^{\dagger}_{k\uparrow} = \left[u_k^* u_k + v_k^* v_k\right]\hat{c}_{-k\downarrow} = \hat{c}_{-k\downarrow}.$$

~~Re~~ $\boxed{\langle \hat{c}_{-k\downarrow}\hat{c}_{k\uparrow}\rangle}$

Look at

$$\langle \hat{c}_{-k\downarrow}\hat{c}_{k\uparrow}\rangle = \langle (u_k\hat{\gamma}_{-k\downarrow} + v_k^*\hat{\gamma}^{\dagger}_{k\uparrow})(u_k\hat{\gamma}_{k\uparrow} - v_k^*\hat{\gamma}^{\dagger}_{-k\downarrow})\rangle$$

$$= \langle u_k^2 \hat{\gamma}_{-k\downarrow}\hat{\gamma}_{k\uparrow} - u_k v_k^* \hat{\gamma}_{-k\downarrow}\hat{\gamma}^{\dagger}_{-k\downarrow} + u_k v_k^* \hat{\gamma}^{\dagger}_{k\uparrow}\hat{\gamma}_{k\uparrow} - v_k^{*2}\hat{\gamma}^{\dagger}_{k\uparrow}\hat{\gamma}^{\dagger}_{-k\downarrow}\rangle$$

(*)

The first term yields: 0 due to annihilation as given in assignment

The second " " : $-u_k v_k^*(1-f(E_k))$ ← (Annett P142)

The third " " : ~~0 due to annihilation~~ ~~$u_k v_k^*(1-f(E_k))$~~ 0 due to annih.

The fourth " " : 0

~~Should~~ yield:

$$\langle \hat{c}_{-k\downarrow}\hat{c}_{k\uparrow}\rangle = -u_k v_k^*(1-f(E_k)) = -u_k v_k^* \tanh\left(\frac{E_k}{2k_BT}\right)$$

minus Annett

$$\Rightarrow \Delta = -V\sum_k \langle \hat{c}_{-k\downarrow}\hat{c}_{k\uparrow}\rangle = V u_k v_k^* \tanh\left(\frac{E_k}{2k_BT}\right)$$

as desired

19 b $\quad u_k v_k^* = \frac{\Delta}{2E_k}$

$$\Rightarrow \Delta = V \sum_k \frac{\Delta}{2E_k} \tanh \frac{E_k}{2k_BT}$$

$$\Leftrightarrow 1 = \frac{1}{2} V \sum_k \frac{1}{E_k} \tanh \frac{E_k}{2k_BT}$$

Changing to an integral, we only need to look at states in $\varepsilon_F \pm \hbar\omega_D$, and have to take into account the degeneracy $g(\varepsilon)$. We take approximately constant around ε_F: $g(\varepsilon) \approx g(\varepsilon_F)$ for $\varepsilon_F - \hbar\omega_D \leq \varepsilon \leq \varepsilon_F + \hbar\omega_D$.

As $E_k = \sqrt{\varepsilon^2 + |\Delta|^2}$, we thus find

$$1 = \frac{1}{2} V \int_{\varepsilon=-\hbar\omega_D}^{\hbar\omega_D} g(\varepsilon) \frac{d\varepsilon}{\sqrt{\varepsilon^2 + |\Delta|^2}} \tanh \frac{\sqrt{\varepsilon^2 + |\Delta|^2}}{2k_BT}$$

$$= V g(\varepsilon_F) \int_{\varepsilon=0}^{\hbar\omega_D} \frac{d\varepsilon}{\sqrt{\varepsilon^2 + |\Delta|^2}} \tanh \frac{\sqrt{\varepsilon^2 + |\Delta|^2}}{2k_BT}.$$

as expected. ~~Ik~~ I used that ~~the integral is even~~ the function under the integral is even and that $g(\varepsilon) = g(\varepsilon_F)$.

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20a Lets look at only the last term

$\cancel{C = \frac{\hbar^2}{2m_e d^2} |\psi_n - \psi_{n-1}|}$

$$G := \sum_n \int \frac{\hbar^2}{2m_e d^2} |\psi_n - \psi_{n-1}|^2 da$$

We write $\psi_n = |\psi_n| e^{i\theta_n}$

and $\psi_{n-1} = |\psi_{n-1}| e^{i\theta_{n-1}}$

as they are complex, and recognize that $|\psi_n| = |\psi_{n-1}|$ for all n due to symmetry of the system between layers (translation should keep it the same), as ψ_n is macroscopic

$\Rightarrow |\psi_n - \psi_{n-1}|^2 = |\psi_n|^2 |e^{i\theta_n} - e^{i\theta_{n-1}}|^2$

$\Rightarrow$ term relates to phase diff

Now look at a Josephson junction:

$$\Delta E = \int_{t=t_1}^{t_2} IV\, dt \quad \text{with} \quad I = I_j \sin\varphi, \quad V = \frac{\hbar}{2e}\frac{\partial \varphi}{\partial t}$$

$$= \int_{t=t_1}^{t_2} I_j \sin(\varphi) \cdot \frac{\hbar}{2e}\frac{\partial \varphi}{\partial t} dt$$

$$= \int_{\varphi=\varphi_1}^{\varphi_2} I_j \sin(\varphi) \frac{\hbar}{2e} d\varphi$$

Now take $\varphi(t_1) = 0$ as initial phase, $\varphi(t_2) = \varphi_f$ as some finite final phase.

$$= \int_{\varphi=0}^{\varphi_f} I_j \sin(\varphi) \frac{\hbar}{2e} d\varphi$$

$$= \frac{I_j \hbar}{2e} \left[-\cos(\varphi)\right]_{\varphi=0}^{\varphi_f} = \frac{I_j \hbar}{2e}\left[1 - \cos(\varphi_f)\right]$$

φ_f relates to the phase difference between layers wave func.

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20 b) ~~If~~ ψ becomes continuous for $d \to 0$, as then the wave functions overlap extremely.

Again, look at the last term:

$$\lim_{d \to 0} \frac{\hbar^2}{2m_c d^2} |\psi_n - \psi_{n-1}|^2 = \lim_{d \to 0} \frac{\hbar^2}{2m_c} \left(\frac{\psi_n - \psi_{n-1}}{d} \right)^2$$

$$\approx \frac{\hbar^2}{2m_c} \left| \frac{\partial \psi}{\partial z} \right|^2$$

Which is of the same form as the third term of F, with different mass. If $m_{ab} = m_c$, the normal G-L form is found (without $\vec{A}$ ~~$\hbar$~~ and $\vec{B}$!

21 For BCS,

$$\hat{H} = \underbrace{\sum_{k\sigma} \varepsilon_k \hat{c}^\dagger_{k\sigma} \hat{c}_{k\sigma}}_{\cancel{H}} + \underbrace{\frac{1}{2} \sum_{kk'} V_{kk'} \hat{c}^\dagger_{k'\uparrow} \hat{c}^\dagger_{-k'\downarrow} \hat{c}^\dagger_{k\uparrow} \hat{c}_{k\uparrow}}_{:= \cancel{H}\ \hat{V}.}$$

Now we choose to $V_{mag} = \frac{1}{2} \sum_q J_q [\hat{\vec{S}}_q \cdot \hat{\vec{S}}_q]$ for our interaction potential

Look at ~~$\hat{\vec{S}}_q$~~ ~~$\hat{S}_q \hat{S}_q \hat{c}^\dagger_q \hat{c}^\dagger_{-q}$~~

$\hat{\vec{S}}_{-q} \hat{\vec{S}}_q \hat{c}^\dagger_q \hat{c}^\dagger_{-q}$ by using $(\hat{S}_1 + \hat{S}_2)^2 = \hat{S}_1^2 + \hat{S}_2^2 + 2\hat{S}_1 \hat{S}_2$ for $[\hat{S}_1, \hat{S}_2] = 0$

$$= \frac{1}{2} [-\hat{S}_{-q}^2 - \hat{S}_{q}^2 + (\hat{S}_q + \hat{S}_q)^2] \hat{c}^\dagger_q \hat{c}^\dagger_{-q}$$

We know that $\hat{S}_i |\psi\rangle = s(s+1)|\psi\rangle$, and here $s = 1/2$

$= \frac{1}{2} \cdot 3/2 |\psi\rangle = 3/4 |\psi\rangle$

Write $J_{k-k'} = \dfrac{J_{k-k'} + J_{k+k'}}{2} + \dfrac{J_{k-k'} - J_{k+k'}}{2}$